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APPLYING FUZZY MATHEMATICS TO RISK ASSESSMENT IN INSURANCE INDUSTRY

SCIENTIFIC PAPER

Abstract

As system structures are complex, and non-linearity, uncertainty, coincidence, variability, etc. occur, most dynamic systems are extremely difficult to model according to accurate and precise mathematical relations. Fuzzy mathematics (fuzzy logic, theory of fuzzy sets, etc.) can provide an appropriate alternative to exact mathematical modelling of systems or processes that are largely vague or too complex to be described by simple and clear mathematical formulas. Methods of the fuzzy system have been widely used in risk assessment and other segments of the theory and practice of insurance, such as classification, underwriting, projected liabilities, future and present value, pricing, asset allocation, cash flows, investments, and more. This paper gives an overview of the research and the possibility to apply fuzzy mathematics in insurance industry.

***Key words**: risk, fuzzy mathematics, insurance, actuarial, underwriting*

1. Introduction

In insurance industry, most of the traditional actuarial methodologies and models are developed based on the theory of probability, mathematical statistics and mathematics. Global competition of the last two decades has opened the door and

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created the need for new methodologies, among them being the methodologies based on mathematics (fuzzy set theory, fuzzy logic). The use of the theory of fuzzy sets is becoming an alternative approach when modelling uncertainty in a stochastic environment. In probabilistic models, it is assumed that the uncertainty in relation to the actual result is estimated based on the performance of the experiment, or the observation of the history of events. The application of fuzzy sets is a better solution for situations when, even after the experiment is performed, we cannot completely present or solve all uncertainties. In practice, this often raises the question of correctness of how to set up the problem and of the subjective interpretation of the results of the experiment. In addition, in insurance practice, there is often a high level of complexity of the observed problem when instead of precise calculations, the experience of human expertise is taken into account.

It turned out that the language of classical mathematics is not sufficiently elastic for modelling where elements or classes of objects do not have a precisely defined criterion for membership in a particular category. Apart from objective laws, subjective assessments or factors play an important role in their functioning. Secondly, by separating the subsystem from its system, we introduce boundaries where no boundaries can be found, so that the transition from one system to the other is abrupt instead of being continuous.

The goal of fuzzy mathematics is to model uncertain or inaccurate, or too complex, or insufficiently defined descriptions that cannot be analysed or are less suitable to be analysed by probability theory or interval mathematics. Its significance and great potential lies in the fact that human thinking by nature is approximate. The approach of gradual and light transitions (recognition of the situation) corresponds to how the fuzzy mathematics approaches the problems and how it resolves them.

The theory of the fuzzy system makes sense in the following situations:

- When the data values and relationships that exist between them are uncertain or imprecise and their assessment is based on the subjective evaluation of the expert;
- When it is difficult to determine the measure of data, or when there are insufficient data from the records necessary for quantitative analysis;
- When some values within the problem are unclear and imprecisely defined;
- The expert's knowledge of the addressed problem is complex, limited and incomplete;
- Where conditions constantly change, so that it is impossible to stochastically describe the analysed value.

2. Risk Concept and Definition

In daily communication, risk is a harmful event that may occur, but it is not certain if it will occur. Harmful events are considered events that have negative consequences on people, society, the economic environment and the natural environment. Many authors describe the concept of risk as the possibility that a particular incident may leave consequences for the accomplishment of the set goal. The risk can be characterized as a complex value that at the same time describes the probability of occurrence of harmful events and the expected magnitude of the consequences of such events in the independent system and during the determined length of the time interval, or during a particular process.

According to the classic approach, risk *(R)* is the product of the probability of occurrence of a certain (harmful) event *(P)* and the expected consequences *(C)*. In complex systems, the total risk can be presented as the sum of all known risks, in such a way that the outcome of risk (or dimension of risk) is comparable (economic loss, loss of reputation, non-material damage, etc.). The total risk of a particular system or process R_U, consisting of a number of individual risks, can be expressed as the sum of individual risks ($i = 1,2, ..., n$) with the probabilities of the realization of *Pi* and the consequences of Ci and can be defined by the following expression[3]:

$$R_U = \sum_{i=1}^{n} R_i = \sum_{i=1}^{n} P_i C_i$$

There are many definitions of risk that describe it depending on the approach, context, and point of view. In the theory and practice of business systems, the concept of risk is most commonly used to express the possibility of the occurrence of an undesired event, taking into account the extent of such an event or the measure of uncertainty. *Knight*[4] defines the probabilities that are obtained by two different methods: theoretical or *a priori* probability based on knowledge of a certain phenomenon, and statistical probability that is generated on the basis of analysis of homogenous statistical data. The above theoretical definition of the concept of risk implies the calculation of the statistical probabilities of various negative events and assessment of the expected consequences of such events. This author also defines the risk as a complete knowledge of the potential solution of a particular situation, that is, the objective probability of the occurrence of such a situation, as well as the consequences. Risk means "to be exposed to the probability of a bad outcome".[5] Some authors consider the risk and uncertainty as two completely independent concepts, while others think that these terms are very dependent. The notion of uncertainty indicates that there is a spectrum of events that can happen in the future, but without indications that they will really happen or that they will leave relative consequences. On the other hand, the risk means that there is a possibility to list

[3] Gordana Radivojević, Vladimir Gajović, "Supply chain risk modeling by AHP and FAHP methods", *Journal of Risk Research*, 17 (3), 2014, pp. 337–352.

[4] Frank H. Knight, *Risk, Uncertainty and Profit,* Boston and New York: Houghton Mifflin, Boston and New York. Reprinted by London School of Economics and Political Science 1948.

[5] Dan Borge, *The book of risk,* John Wiley & Sons, New York, 2001.

events that can happen in the future, and that they can individually determine the likelihood of realization. The concept of uncertainty is also described as the absence of information in the conditions when decisions are made and implies the assessment of a particular situation, alternative solutions, possible results and consequences, and the like. In extreme situations, uncertainty can be characterized as the absence of information or knowledge about a particular problem or in the decision-making process. In the area of risk management, there is integration and interaction of terms *uncertainty* and *risk*, which is usually combined with the notion of risk.

3. Statistical Methods for Risk Assessment

Different methods for risk assessment are used in insurance practice. They are most precisely based on the law of large numbers, that is, on statistical parameters and a representative statistical sample from the historical database that relates to the observed event. Additionally, in theory and practice of the assessment of the risk in business systems, as well as in the field of insurance, statistical data are often combined, and they are based on appropriate statistical assumptions that are not confirmed by the exact principles of the theory of mathematical statistics. However, if all the observed risk parameters are based on a representative statistical sample, a high reliability of the statistical assessment is achieved.

According to *the American Academy of Actuaries*,[6] statistical considerations of risk include the following:

- *Homogeneity*. Homogeneity of risk represents equality or similarity of the characteristics of risks belonging to the same class. Homogeneous risks can be viewed as a unique set when forming an appropriate statistical sample in the process of risk analysis and quantification.
- *Credibility.* A general statistical principle is that the larger the number of observations, the more accurate are the statistical predictions that can be made. The repetition and the number of events are the parameters that are required in determining certain principles governing the actuation of an event. It is very rare that risks that occur cannot be assessed by statistical methods. Instead, alternative, non-statistic risk assessment methods can be used.
- *Predictive stability.* A major consideration in the construction of risk classification systems is its predictability i.e. unchangeability in the future period. Predictive stability of risk behaviour implies that its characteristics will not change significantly, thus ensuring the accuracy of statistical evaluation. However, one of the basic characteristics of risks, especially in dynamic

6 *American Academy of Actuaries, Committee on Risk Clasification*. 1980. Risk Classification Statement of Principles.

systems, is that they change in space and time due to various internal or external causes. There are many risks in business systems and processes that cannot be considered stable. Due to the instability of certain parameters of business systems and processes, it is often not possible to provide predictive stability as a criterion for statistical risk assessment.

When precisely measuring the level of insurance risks, the main problem lies in the fact that some risks occur very rarely. In addition, some risks may be represented in a business system, however, until the moment of their assessment they have not been realised in practice (e.g. fire risk, explosion risk, flood risk, etc.). If only one year is selected for the limited time in which the risk is monitored, in that period the observed risk does not have to be realized or it can be realized only once. This would mean that many years need to pass for a sufficient number of input parameters for the assessment of the specific risk to be formed. This is often impossible to implement in practice. Also, the observed risk can change with the passing of time or, for example, leave a number of other accompanying consequences that are cumulative in relation to primary risk (e.g. loss of earnings and reputation, loss of market, delays in production, additional costs, immaterial damages etc.).

Actuaries are highly educated experts who possess knowledge in general mathematics, mathematical statistics, stochastic processes, probability theory, financial and actuarial mathematics, finance, financial management and investment in insurance, insurance, accounting, demography and other. With the help of actuaries, technical bases, more precisely, calculation models for individual branches of insurance are set up - the insurance premium is determined, the income and expenditure accounts of individual classes are prepared and planned, the results are monitored and analysed in order to verify the accuracy of the set mathematical models and, finally, of total business from the standpoint of insurance industry.

In actuarial practice, various problems relating to the reliability of risk assessment are solved by applying *the Credibility Theory*.[7] Credibility is a measure of a predictive value that is assigned to a particular set of data, or a measure of statistical reliability, confidence, or probability. The credibility theory is, in certain cases, applied when defining insurance premiums or modelling complex insurance processes, and is most often based on appropriate estimates of the credibility factor value. If there is not a sufficient number of statistical data in the risk assessment of homogeneous groups of insurance subjects, the accuracy of the assessment will not be sufficiently credible.

Actuarial science involves the design and evaluation of future financial parameters and events based on valid statistical data form the past, using actuarial methods and models, or by individual subjective evaluation of certain financial

[7] Robert Brown, Leon Gottlieb, *Introduction to Ratemaking and Loss Reserving for Property and Casualty Insurance.* 2nd Edition. ACTEX Publications Inc., 2001.

parameters, in conditions where there are no representative data from the past. The accuracy and reliability of any actuarial analysis depends on sufficiently available statistical data, in a certain time span as well as in the historical period, and on their reliability and stability in time. However, there is a steady increase in demand for good statistics for the purpose of advanced risk analysis in order to support insurance activities, as well as processes of risk assessment and reduction of potential damages. Using various types of statistical tools, actuaries analyse past data to determine the insurance premium, analyse the need to change premium rates for different services, project provisions, or other actuarial indicators. Defined premium rates or insurance premiums are further considered and corrected in other insurance functions, especially in conditions of low credibility of the data, when the risk is most often assessed individually, by experts, due to the fact that it is not possible to fulfil the above actuarial assumptions. Underwriters are among the most important experts, whereas the segment of underwriting in insurance is one of the most important activities of the insurer in the function of providing the appropriate insurance coverage. Underwriting entails the process of analysing, selecting and classifying claims for insurance, assessing the exposure of potential clients to certain risks, and determining the terms and prices of insurance coverage. Underwriting involves the process of analysis, selection and classification of insurance applications, evaluation of potential exposures of customers to particular risks, and definition of conditions and price of the insurance cover.[8,9] Underwriters are professionals who have the necessary knowledge, experience and intuition in terms of risk and insurance cover and are often closely specialized in certain lines of insurance. In addition to a good knowledge of certain insurance services, underwriters must have the necessary knowledge in the field of reinsurance, risk management, claims settlement and management of claims and recourses in insurance, actuarial science, economics and finance, engineering and technological sciences, law, etc.

Today, many methods and techniques for multiple criteria decision making, based on fuzzy logic, are in widespread application in various fields such as management, economics, technical sciences, etc., including the field of insurance underwriting.

4. Methods and Models for Insurance Risk Assessment Based on Fuzzy Mathematics

The assessment of insurance risks depends on the complexity, perception of individuals, the impact on the system, the availability of information and other circumstances. In the event that there is a lack of representative statistical data,

[8] Lionel Macedo, *The Role of Underwriter in Insurance*, Primer Series of Insurance, The World Bank, 2009.

[9] Robert Klen, "Underwriting Cycle", *Encyclopaedia of Actuarial Science,* Vol. 3, (Editors-in-Chief, Teugels J., Sundt B), 2004, pp. 1705–1714.

available statistical data are often used in practice, with certain reservations, limitations and deviations. These data are based on the intuition or knowledge of experts. By applying the theory to fuzzy system, satisfactory solutions can be obtained in cases where there is a problem of uncertainty, unreliability, ambiguity and vagueness.[10] In the insurance risk assessment segment, it is used when there are no clear, precise or sufficient data necessary for reliable risk quantification. Fuzzy systems and fuzzy technologies represent a mathematical approach based on which certain imprecise information can be mathematically modelled, which is the basis for computer information processing by numerous software in this field. This provides adequate support in deciding on the solution of various problems, and one of the possibilities of application is the field of risk analysis and assessment. The classical theory often represents an inadequate basis for the analysis of relationships or events based on linguistic variables, such as, for example, *very high risk, low level of safety*, *highly flammable matter, etc*. On the other hand, fuzzy logic provides an inference morphology that enables the application in knowledge-based systems or processes, and allows the formalization of the approximate human reasoning. The nature of human behaviour is to reason on the basis of evidence, which is the basis for making appropriate decisions and achieving goals. However, uncertainty and vagueness are the most common reasons for the occurrence of errors in assessing the characteristics and magnitudes of certain phenomena, as there is often no clear and accurate information about the environment. Most dynamic, physical systems, especially complex systems, are extremely difficult to model according to accurate and precise mathematical relations, due to the complexity of system structure, non-linearity, uncertainty, coincidence, variability, etc. Fuzzy mathematics can provide an appropriate alternative to exact mathematical modelling of different dynamic systems that are, to a great extent, unclear or too complex to be described in simple and clear mathematical formulas.

Today, there is a common belief that, when a complex physical or control system does not provide a set of appropriate equations as a precise or acceptable mathematical model, the fuzzy systems theory and control based on it have some acceptable traits and distinctive characteristics in relation to other approaches. Compared to conventional approaches, fuzzy control systems are based more on information from the domain of expertise, and less on mathematical modelling of the physical or other system.[11]

In the theory of classical sets, a certain set is a collection of things or events that can be differentiated as individual elements that share some common traits. Each

[10] For more about this area refer to u Lotfi A. Zadeh, "Fuzzy sets", *Information and Control,* (8), 1965, pp. 338–353. and Hans-Jiirgen Zimmermann, *Fuzzy Set Theory and its Applications*, 4th Edition, Springer, 2001.

[11] Chen Guanrong, Tat Pham Trung, *Introduction to Fuzzy Sets, Fuzzy Logic and Fuzzy Control Systems*, CRC Press LLC, 2001.

element is individually called a member or element of a set. If the sets are marked with capital letters *A, B, C*..., and elements of set are marked with small letters *a, b, c*..., so that the individual element *a* belonging to the set *A* is marked with $a \in A$. The symbol „ $\in$ " is read as „*element of*", and the fact that the element *a* is not a member of the set *A* is written with $a \notin$ A. According to the theory of ordinary (crisp) sets, there are only two possible relations between individual element *a* and the set *A;* that is, either $a \in A$ or $a \notin A$. The universal set consists of all individual elements in the given application.

In literature, fuzzy sets are defined in many ways, because depending on the set of values of the corresponding membership function, there are several types of fuzzy sets. In the basic type of the set, the membership function has values in the interval [0, 1]. If X is an arbitrary non-empty set, fuzzy set A with values in the interval [0, 1] defined on X is characterized by a function $\mu_A : X \rightarrow [0,1]$, that is, an arranged pair (X, μ_A) is called a fuzzy set. The function μA is called the membership function of the fuzzy set A. The membership function μ_A (x) is often also denoted by A (x), and is identified with the fuzzy set (X, μ_A). The value μ_A (x) is interpreted as the degree of membership of element x to the set A.

When X is continuous, the fuzzy set A is mathematically written as $A = \int_X \mu_A(x)/x$, which denotes a collection of all points $x \in X$ with the corresponding membership function μ_A (x). On the other hand, when X is a discrete set, fuzzy set A is usually expressed as $A = \sum_X \mu_A(x)/x$, which denotes a collection of all points $x \in X$ with the corresponding function of μ_A (x). It is desirable to define the operations with fuzzy sets so that good characteristics of the ordinary sets and operations can be applied to the fuzzy sets, to the greatest possible extent.

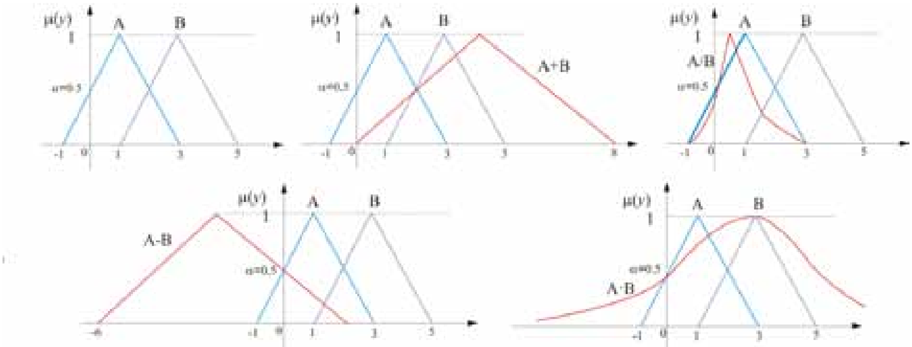
The fuzzy arithmetic is based on the characteristics that each fuzzy set, and therefore the fuzzy number, can be completely and uniquely represented by using the α -cut i.e, alpha-cut, and that the α - cut of the fuzzy number is a closed interval of real numbers for all $\alpha \in [0,1]$.

If * denotes any of the four standard arithmetic operations +,-,. and /, analogous operations may also be defined with closed intervals, when $[a,b]*[d,e] = \{ f * g | a \le f \le b, | d \le g \le e \}$ except when dividing, which is not defined if $0 \in [d,e]$. Main arithmetic operations with fuzzy numbers are defined through α -cut. Most commonly used fuzzy numbers are the triangular fuzzy numbers.

Image 1 shows basic arithmetic operations with triangular fuzzy numbers $A = [-1,1,3]$ and $B = [1,3,5]$. [12]

[12] *Adapted,* George J. Klir and Bo Yuan, *Fuzzy sets and fuzzy logic, theory and applications*, Prentice Hall, New Jersey, 1995

Figure 1. Main arithmetic operations with triangular fuzzy numbers



The models based on fuzzy logic consist of *if-then* rules. The known value of the input variable x needs to pass through all the defined rules, i.e. it is necessary to determine how much truth there is in the assertion that a certain parameter (e.g. x) has a certain value, e.g. *small, medium or large value.* After going through all the rules in the approximate reasoning algorithm, each of the possible values of the output variable (for example y) is associated with the corresponding grade of membership. The last step in the approximate reasoning algorithm is defuzzification, i.e. choosing one value for the output variable. After analysing the grade of membership of individual values of the output variable, the decision maker chooses one value of the output variable.

Methods of the theory of the fuzzy sets and fuzzy logic have also been widely used in the assessment of risks and other segments of the insurance theory and practice, such as classification, risk acquisition, projected liabilities, future and present values, premium setting, asset allocation, cash flows, investments and other. Very often, especially in smaller insurance companies, there is a problem of providing representative statistical parameters necessary for precise risk quantification. In addition, in practice, the question is often raised as to the correctness of the methods and procedures for structuring the analysed problem and of the subjective interpretation of the experiment results.

First basic definition was presented by applying fuzzy logic in insurance to provide a flexible definition of a preferred policyholder in life insurance.[13] The following fuzzy procedure is illustrated in the reinsurance models, and then the theory of fuzzy numbers is expanded to define the fuzzy insurance premium. Fuzzy sets have recently been applied in actuarial science.

[13] Jean Lemaire, "Fuzzy Insurance", *ASTIN Bulletin*, 20 (1), 1990, pp. 33–55.

Zimmerman[14] provides a comprehensive overview of the application of the fuzzy sets, including the treatment of uncertainty due to the lack of information that generates risks in the decision-making process. *Ostaszewski* proposed a large number of possible applications of fuzzy theory in actuarial analysis. His research included economic risk, time value of money, individual and collective risk models, classification of assumptions, conservatism and adjustment.

Cummins and *Derrig*[15] supplemented previous works with their research of the application of fuzzy sets to the predictions in property insurance and liability insurance, and to the problem of determining the premium. *DeWit*[16] explicitly stated that the insurance contracting process consists of the risk and uncertainty that classic theories of probability cannot accurately describe. This author also mentioned the limitations in the application of the fuzzy set theory, however, he considers that in the areas where classical risk theory offers insufficient possibilities, there is room for calculating insurance premiums based on this theory. *Shapiro,* in his papers[17], provides the overview of broad possibilities in the application of fuzzy logic in insurance in the segments of underwriting, risk assessment and determination of insurance premium, risk classification, projections of insurer's liabilities arising from insurance contracts, investments, allocation of financial resources, etc.

A number of studies have been devoted to the research of risk classification and the application of the theory to fuzzy sets for the classification of insurance elements such as life insurance risks, risk and loss classification, classification of sources of misuse and fraud, classification of the risk of work-related injuries, disability index classification, as well as many others, because the risk classification represents an introduction to the potential assumption of insurance risks that need to be properly identified, classified and separated in order to be priced. *Ostaszewski*[18] stressed that the classification of insurance risks often resorts to rather vague and uncertain methods such as *high risk* or *low risk areas*, or methods that are excessively precise – as in the case of a person who may fail to classify as a preferred risk because his body weight exceeds the specified limit. *Lemaire* said that the lack of actuarially fair classification is economically equivalent to price discrimination in favour of high-risk individuals and suggested a possible precaution against (discrimination) which is to create a classification method with no assumptions, but rather methods which uncover patterns used for classification.[19] He was among the first authors to suggest

[14] Hans-Jiirgen Zimmermann, *Fuzzy Set Theory and its Applications*, 4th Edition, Springer, 2001.

[15] David Cummins, Richard A. Derrig, "Fuzzy Financial Pricing of Property-Liability Insurance", *North American Actuarial Journal*, 1 (4), 1997, pp. 21–40.

[16] G. W. DeWit, "Underwriting and Uncertainty", *Insurance: Mathematics and Economics* 1/1982, 277–285.

[17] Arnold Shapiro, 2004. 2005, 2007.

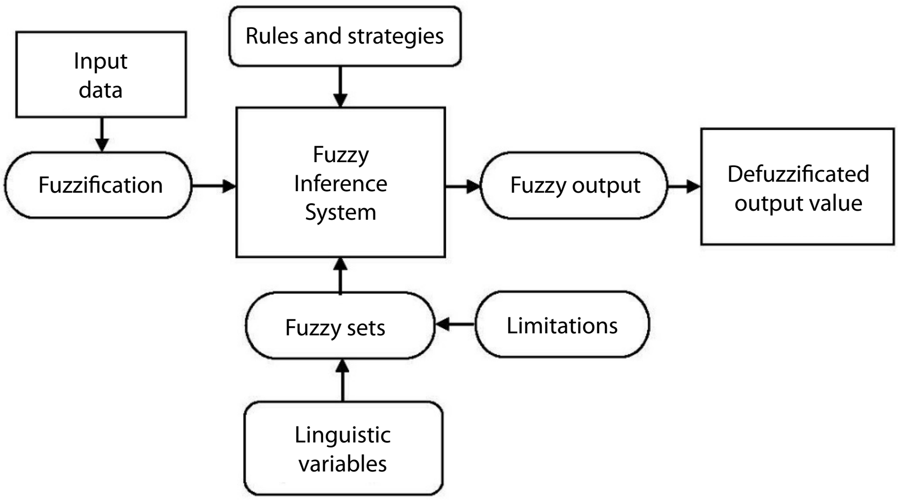
[18] Krzysztof Ostaszewski, *Fuzzy sets Methods in Actuarial Science*, Schaumburg IL, Society of Actuaries, 1993.

[19] Lemaire, J. Ibidem, str. p. 35.

the use of *c* – means algorithm for the classification in insurance. When presenting his study to the actuaries, he discussed the example involving the classification of four prospective insureds, two men and two women, in two clusters, based on the height, gender, weight and resting pulse. Two initial clusters were based on the gender. In phases, and after three repetitions, *Ostaszewski* developed a more efficient classification based on all the traits of the insureds.

Risk modelling is a dynamic process consisting of a wide range of activities and skills, including system or process analysis, development, testing, simulation and application of methods and models and periodic improvements and corrections.[20] In the recent researches, integrated fuzzy mathematical methods and fuzzy expert systems are increasingly used for the purpose of best risk assessment. There are numerous researches in the area of risk management and different methods and models for solving a number of problems related to risk management. An appropriate model needs to be implemented in the fuzzy inference system, in order to obtain output values within predetermined range.

Figure 2. Fuzzy inference system



Source: Adapted, Shepard, R. 2005. Quantifying Environmental Impact Assessments Using Fuzzy Logic, Springer.

[20] Vladimir Gajović, Marija Kerkez, Jelena Kočović, "Modeling and simulation of logistic processes risk assessment with fuzzy logic technique", *SIMULATION: Transactions of the Society for Modeling and Simulation International*, 2017, pp. 1–12.

Figure 2 shows general concept of fuzzy inference system. Numerical values of the observed parameters are firstly fuzzified. Then, taking into account different limitations, linguistic variables (that is, fuzzified values) enter in the set of fuzzy laws and activate one of them. Subsequently, fuzzy inference system is implemented and as a result, a defuzzificated output value is obtained in the form of a linguistic variable or a certain quantitative value.

Lemaire suggested the fuzzy logic methodology for underwriting of individual risks, as well as fuzzy calculation of premiums and insurance reserves. In his agenda, he showed a wide application of fuzzy logic in the insurance theory, notably from the finance perspective.[21] He used a fuzzy expert system to give a flexible definition of a desired insured in life insurance. As part of these efforts, he used continuous membership functions, expanded the definition of the intersection to include limited difference, *Hamacher* and *Yager* operators, and showed how α-cuts may be applied to make selections in the decision rule where α-cuts are applied to each membership function and algebraic product, whereby minimum acceptable product equals α-cut. *Lemaire* focused on the following features: cholesterol, blood pressure, weight and status of a smoker, and their intersection.

Following *Lemaire's* lead, *Hosler*[22] and *Young*[23] used fuzzy expert systems to create the selection process in group health insurance. *Young* suggested the method of applying fuzzy sets in group underwriting in health insurance and gave the algorithm applied to group health insurance and underwriting with the help of 13 fuzzy rules for their acceptance. Firstly, single-plan underwriting was considered, and then, the study was expanded to the plans with more options. In the single-plan situation, *Young* focused on such fuzzy input features as change in the age/sex factor in the previous two years, change in the group size, proportion of employees selecting group coverage, proportion of premium for the employee and a dependent paid by the employer, claims as a proportion of total expected claims, the loss ratio adjusted for the size of an insurance company. She completed the section with a discussion of a matrix of the interaction between the features (criteria) and their interpretation in the context of fuzzy intersection operators.

In the multiple-option case, the additional fuzzy features include single and family age factors, desired participation in each plan, age/sex factors, the difference in the cost of each plan, and the relative richness of each plan. The age factors depended on the possibility of participation, given access cost, the richness of the benefits, employee cost, marital status, and age. The underwriting decision in this case included the single-plan decision as a criterion.

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[21] Jean Lemaire, "Fuzzy Insurance", *ASTIN Bulletin*, 20 (1), 1990, pp. 33–55.

[22] Virginia R. Hosler, "The application of fuzzy sets to group health underwriting" *ARCH* 2 /1992, pp. 1–63.

[23] V. R. Young, 1996. Insurance Rate Changing: a Fuzzy Logic Approach, *Journal of Risk and Insurance*, 63 (3), 461–484.

Carreno and *Jani*[24] developed a knowledge-based system that combines fuzzy processing with a rule-based expert system in order to provide an improved decision aid for evaluating life insurance risks. Their system used two types of inputs: the base inputs age, weight and height; and incremental inputs which deal with particular habits and characteristics of prospective clients. The output of their system was a risk factor used to develop the premium surcharge.

McCauley-Bell and *Badiru*[25] suggested fuzzy linguistic expert system for quantifying and predicting risks against the occupational injury of forearm and hand. The first phase of research focused on the development and representation of linguistic variables for the purpose of defining the risk levels. These variables were subsequently quantified using fuzzy sets. The second phase used analytic hierarchy processing to assign relative weights to the identified risk factors. Using linguistic variables obtained in the first part of the research, a fuzzy rule base was constructed with all of the potential combinations for the given factors. The basic characteristic of their approach is the use of concept mapping to capture a detailed representation of the expert's knowledge relative to the problem space, as well as an understanding of the relationships between concepts. Subsequently, the study was expanded[26] by developing a fuzzy linear regression model to predict the relationship of known risk factors to the onset of occupational injury. Following *Tanak* at al.,[27] their model took over a general form:

$$\tilde{Y} = \tilde{A}_0 + \tilde{A}_1 x_1 + \ldots + \tilde{A}_n x_n$$

where $\tilde{Y}$ is fuzzy output, $\tilde{A}_i$ is symmetrical triangular fuzzy number, and $\boldsymbol{x} = (x_1, \ldots, x_n)$ is *n*-dimensional non-fuzzy input vector. The main idea, which is often called fuzzy regression approach, was to diminish the fuzziness of the model by minimizing fuzzy coefficients, according to the inclusion of items (data) for each sample in a particular α-cut. Their model was constructed using four fuzzy sub-models to represent the risk associated with task, anthropometrics, joint deviation and personal factors. These models were combined to produce an overall risk level and a final risk prediction model. The results show that fuzzy linear regression is a useful technique for addressing uncertainty associated with definition and modelling of the occupational injury risk factors.

24 Enciklopedija osiguranja

25 Pamela McCauley-Bell, A.B. Badiru, "Fuzzy modeling and analytic hierarchy processing to quantify risk levels associated with occupational injuries. The development of fuzzy-linguistic risk levels", *Fuzzy Systems, IEEE Transactions on Fuzzy Systems*, Volume: 4/1996, pp. 124–131.

26 Pamela McCauley-Bell, L.L Crumpton, H. Wang, "Measurement of cumulative trauma disorder risk in clerical tasks using fuzzy linear regression", *IEEE Transactions on Systems, Man, and Cybernetics*, Part C, Volume: 29/1999, 1–14.

27 Hideo Tanaka, I. Hayashi, J. Watada, "Possibilistic linear regression analysis for fuzzy data", *Eur. J. Operation Research*, 40/1989, pp. 389–396.

Risk assessment approach in cargo insurance in Serbia is shown in the paper written by the authors *Kerkez* and *Gajović*.[28] Considering the complexity of the analysed problem, number of risks, risk parameters and causes, the lack of representative statistics for numerous risk elements and the overall risk, the authors have shown how FAHP method (*Fuzzy Analytic Hierarchy Process*) can provide acceptable quantitative data.

Models based on fuzzy logic usually require iterations. The set of rules and corresponding functions belonging to the input and output variables are firstly defined for the observed problem. Then, the model is tested and, if necessary, correction of certain rules and functions of membership is made, after which the model is re-tested, until satisfactory solutions are obtained. Based on the fuzzy logic, risk parameters may be shown as fuzzy numbers which can be further combined and matched with the defined rules from the established rule base and assessed on the basis of, for example, *Mamdani* inference system. In the last phase, defuzzification is made for the purpose of obtaining a clear quantitative value, for example to assess the level of risk of the insurance object, which is expressed by a premium rate or insurance premium.

4. Conclusion

Fuzzy logic is a formal apparatus for modelling the inference process in the conditions of unclear and inaccurate information in a business system or a process. In situations where, even after a thorough analysis of the available statistical data, all uncertainties cannot be fully presented or resolved, fuzzy systems can in certain cases be a better solution than stochastic methods. System theory allows the use of subjective assessments expressed through uncertain concepts, relations and statements for describing problems, choice of alternatives for the decision-making, formulation of unspecified descriptions using fuzzy variables, and presentation of output results using linguistic terms and relations, or in the form of clear quantitative recommendations. The availability of information on a number of risk parameters for specific insurance, collected from a very limited past period, affects the reliability of the assessment of the risk of insurance coverage. In certain cases, it is possible to access a statistical database on realized risks, however, the data are usually not representative. For this reason, knowledge, intuition and experience of experts are often used in insurance practice for the purpose of risk analysis, assessment and control. Fuzzy logic can be one of the most effective methods for risk analysis and

[28] Marija Kerkez, Vladimir Gajović, *Underwriting risk assessment in marine cargo insurance, Risk management in the financial services sector*, University of Belgrade, Faculty of Economics Publishing Centre, vol. 1, No. 1/2016, pp. 381–399.

assessment in different areas. It offers great potential for risk insurance modelling, especially in the initial stages of defining the concept of risk, where information on the reliability of elements or processes, as well as risks, is scarce. Fuzzy logic can find significant application especially in types of insurance in which formalized premium rates are not applied in practice (e.g. in certain types of transport and credit insurance lines, different liability insurance lines, specific property insurance lines etc.), or the types where the scope of insurance coverage and premium rates are assessed on the basis of intuition, experience, and knowledge of risks. This approach can provide highly acceptable solutions to risk assessment problems, which traditional methods are not able to find.

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