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# ADJUSTMENT OF MORTALITY TABLES BY LIMITED FLUCTUATION METHOD

SCIENTIFIC PAPER

**Abstract**

Credibility theory provides tools to deal with the randomness of data that is used for predicting future events. The credibility procedure is used to improve the statistical significance of parameter estimates in a given task. Because it has been used since the very beginnings of actuarial practice, it is one of the most productive actuarial techniques, and is still being developed and adapted to solve contemporary problems. In this paper, the credibility procedure will be used to modify the standard population mortality table, more specifically the Serbian mortality table for females (Mortality Tables of the Republic Statistical Office 2010-2012), to reflect the experience with a particular subset of population and thus establish a more reliable estimate of mortality by including into analysis the experiential data and subgroups.

***Key words:*** *mortality tables, mortality rates, credibility*

## I Introduction

In recent years, regulators and pension funds have been paying greater attention to assumptions about mortality rates used in pension schemes. One of the reasons is an increasing awareness of the variability in mortality rates within different demographic groups and/or populations within the schemes, which results in the need to reconsider assumptions and better adapt to specific plans. Practical evidence suggests that mortality varies by industry, geographical area, and type of business. This has led to a renewed interest in credibility theory as the means of adapting standard mortality tables to specific pension schemes or to the population covered by those schemes. In this regard, in the USA and Canada, it is legally prescribed that the retirement plans use customized mortality tables

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which are prepared by using credibility theory.[3] Guidelines for the use of credibility procedure have been established and the Actuarial Standard of Practice no. 25 (Credibility Procedures) has been extended to include retirement plans.

The concept of credibility has been used in actuarial practice since the beginning of the actuarial science. One of the first recorded works on this subject is the work of Albert Mowbray[4] published in 1914. Early discussions of credibility were focused on estimating the mean incidence of damage using classical and empirical credibility procedures. Later, formulas were developed for Bayesian credibility procedures and limited fluctuation method[5]. The latest credibility methods include credibility estimation for generalized linear models or other multivariate techniques. Credibility procedures involve different approaches. Depending on the problem, an approach may be based on estimation, mathematical models, or their combination.[6]

This paper will explain the method of limited fluctuation and show how to use the credibility procedure when adjusting standard mortality tables to specific retirement plans.

## II Credibility Procedure

Credibility theory provides tools for dealing with random variables (data) that are used to predict future events. Credibility is a measure of the predictive value in a given application that the actuary attaches to a particular set of data (predictive is used here in the statistical sense, and not in the sense of predicting the future).[7] Credibility procedure is a process that involves the evaluation of subject experience for potential use in setting assumptions without reference to other data, or includes the identification of relevant experience. Credibility procedure is also used to improve estimate of the parameter in a given task. Credibility can be used for insurance pricing, calculation of premium rates, determining future premium rates based on experience and reservations, and more.

The insurance company uses past claims data to estimate future costs of insurance coverage. However, insurance claims arise from accidental occurrences. Using the data on average annual claims in the past few years may result in a poor estimate of costs for the coming year. The expected accuracy of this estimate is a

[3] Internal Revenue Service (IRS), www.irs.gov

[4] Albert H. Mowbray, „How extensive a payroll exposure is necessary to give a dependable pure premium? "

[5] Detailed formulas can be found in the papers of Gavin Benjamin, "*Selecting Mortality Tables: A Credibility Approach*", Research paper, Society of Actuaries, 2008 and American Academy of Actuaries, *Credibility Practice Note*, 2008.

[6] See Marija Kerkez, Nebojša M. Ralević, "Uncertainty analysis and risk modelling in insurance", *Insurance in the post-crisis era*, (Kočović, J. Baskakov. V. Boričić, B. et al. eds) University of Belgrade, Faculty of Economics Publishing Centre, 2018, Chapter 18, 309-326.

[7] Actuarial Standard of Practice No. 25. Credibility Procedures Applicable to Accident and Health, Group Term Life, and Property/Casualty Coverage.

function of the variability of claims. These data alone are not acceptable for the calculation of insurance premium rates.

When applying credibility methods, it is necessary to take into account the characteristics of both the company experience (past data) and the relevant experience (experience similar to that of a company). The actuary should use expert judgment when selecting, developing or using the credibility method. When making decisions, attention is paid to the extent to which the subject experience is incorporated into the relevant experience. If the subject experience is an essential part of the relevant experience, the actuary should use his or her expert judgment in deciding if he or she should use that relevant experience and in what way. In addition, homogeneity and predictive stability of these data should be taken into account, whereby segments that are not typical representatives of the experience as a whole can be excluded and thus better predictive value can be obtained. The actuary also considers the balance between the homogeneity of the data and the size of the data set.

The basic formula for calculating credibility weighted estimates is:

Estimate = Z x [Observation] + (1-Z) x [Other information], 0 <Z <1,

where Z is credibility factor assigned to observation, 1-Z is generally called complement of credibility factor.

## 1. Limited Fluctuation Credibility

Classical credibility procedures make assumptions as to the form of the underlying probability distribution. The corresponding number of claims, the amount of premium, etc. are calculated on the basis of this probability distribution, so that the probability of movement of subject claims is within the indicated percentage of expected value. One such approach that assumes that claims follow a normal distribution is Limited Fluctuation Credibility.

Limited fluctuation credibility method is less rigorous than the greatest fluctuation credibility[8], however, it requires subjective estimates of parameters. The method uses a linear combination of relevant experience $\hat{m}$ and subject experience $\hat{\alpha}$ to estimate:

$$\hat{E} = Z \cdot \hat{m} + (1 - Z) \cdot \hat{\alpha},$$

where credibility factor Z needs to be established.

If Z = 1, the data meet the criterion for full credibility, whereas for Z = 0 the analysed data have no credibility for the observed process. If 0 < Z < 1, the observed body of data is not sufficient, and that is a so-called partial credibility.

[8] For more information about this method refer to the papers: Stuart Klugman, Thomas Rhodes, Marianne Purushotham, Stacy Gill, MIB Solutions. *Credibility Theory Practices*, Society of Actuaries, 2009 and Stuart Klugman, Sample size selection for multiple samples-*A brief introduction to credibility theory and an example featuring rate-based insurance premiums*, Drake University, for presentation at NYU-March 4, 2004.

The model is designed to ensure that the error around the experience of the company is minimised to an acceptable level, where the acceptability of the error level is assessed by the actuary. Namely, subjective judgement is required.[9]

In this model, for Z=1 there is full credibility if $Pr\left(\left|\hat{m}-m\right|\leq rm\right)\geq p$, which represents a relative error, where $\hat{m}$ is the estimate based on the subject experience, and m is the true, unknown value. The choice of confidence level (p) and margin of error (r) is subject to judgment. This is one of the main disadvantages of this method. In addition, a subjective judgment is required to determine the quantity of data necessary for full credibility. A 90% confidence error and 5% margin of error (p = 90% and r = 5%) are frequently cited as minimum levels required for full credibility; however, since there is no theoretical basis for this threshold, other assumptions may be just as valid. Thus, a 3% margin of error with 90% confidence level is recommended in Canada for purposes of setting mortality assumptions, and full credibility is achieved at 3.007 deaths. On the other hand, the proposed US mortality regulations define full credibility at 1,082 deaths, which is based on a 5% margin of error with a 90% confidence level.[10]

Let n be the number of observed lives at a given age (x), let d be the number of observed deaths, and let q be the true mortality probability.

Full credibility is assigned if the following is true

$$\Pr\left(\left|\frac{d}{n}-q\right|\leq 0,05q\right)\geq 0,9.$$

If n is large enough, using de Moivre-Laplace theorem, the binomial distribution with $n$ and $q$ or $B(n,q)$ can be approximated by a normal distribution, with the parameters $\mu=nq$ and $\sigma^2=nq(1-q)$, or $N(nq,nq(1-q))$.

$$\Pr\left(0,95nq\leq d\leq 1,05nq\right)\approx\Pr\left(\frac{0,95nq}{\sqrt{nq(1-q)}}\leq z\leq\frac{1,05nq-nq}{\sqrt{nq(1-q)}}\right)\geq 0,9.$$

This formula can be simplified by noting that $1-q\rightarrow 1$

$$\Pr\left(-0,05\sqrt{nq}\leq z\leq 0,05\sqrt{nq}\right)\geq 0,9.$$

For a standard normal random variable $Pr(-1,645\leq z\leq 1,645)=0,9,$ so the equation is satisfied when $0,05\sqrt{nq}\geq 1,645$ or $nq\geq 1.082.$ Thus, the expected number of deaths must be at least 1,082 for the full credibility requirement to be met.

Generalizing this for any p and r leads to the conclusion that full credibility is achieved when the number of observed deaths is greater than or equal to $(z_{(1+p)/2}/r)^2$.

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[9] For more information regarding subjective actuarial estimates refer to Marija Paunović, *Uncertainty Measures and Actuarial Application*, Doctoral Dissertation, Faculty of Technical Sciences, University of Novi Sad, 2019.

[10] Irina Pogrebivsky, Credibility Educational Resource for Pension Actuaries, *Application of Credibility Theory to Mortality Assumption*, Society of Actuaries (SOA), 2017.

In case the full credibility is not met, some portion of credibility is assigned to relevant data.

$$\sigma^2\left(\hat{E}\right)=\sigma^2\left(\hat{m}\right)=\sigma^2\left(\frac{d}{n}\right)=\frac{q\left(1-q\right)}{n^{cr}}\approx\frac{q}{n^{cr}}$$

$$\sigma^2\left(\hat{E}\right)=\sigma^2\left(\hat{m}\right)=\sigma^2\left(Z\cdot\hat{m}+\left(1-Z\right)\cdot\hat{\alpha}\right)=Z^2\sigma^2\left(\frac{d}{n}\right)\approx\frac{Z^2q}{n^a}$$

$$\sigma^2\left(\hat{E}\right)=\sigma^2\left(\hat{m}\right)=\sigma^2\left(Z\cdot\hat{m}+\left(1-Z\right)\cdot\hat{\alpha}\right)=\sigma^2(Z\cdot\hat{m})+\sigma^2((1-Z)\cdot\hat{\alpha})$$

$$=Z^2\sigma^2(\hat{m})+(1-Z)^2\sigma^2(\hat{\alpha})=Z^2\sigma^2\left(\frac{d}{n}\right)\approx\frac{Z^2q}{n^a}$$

where $n^a$ is the number of actual deaths and $n^{cr}$ is the number necessary for full credibility.

Matching the two variances, we obtain

$$Z=\left(\sqrt{\frac{n^a}{n^{cr}}},1\right).$$

This means the credibility factor is the square root of the ratio of the observed (actual) deaths (the expected deaths are not available) to the number of deaths required for full credibility.

## 2. Adjustments of Standard Mortality Tables

Mortality tables are an essential demographic tool for analysing population mortality, and represent a tabular overview of mortality by age and sex. Mortality tables, that are created through observation of the entire population of a particular nation, are commonly referred to as population tables or standard mortality tables. Tables formed on the basis of mortality data derived from a collection of insurance portfolios or certain retirement plans are referred to as market mortality tables.

The credibility procedure is used to modify a standard population mortality table to reflect the experience of a subgroup for which a more appropriate mortality estimate can be determined by including the experiential data for that subgroup. When evaluating a retirement plan, the goal is to include the experiential data of participants covered by the evaluated plan. An employer may have contracts concluded with multiple retirement plans, however, a single plan may include multiple employee subgroups. In such cases, an actuarial assessment is used to decide whether different subgroups require different mortality tables, that is, produce mortality tables for a particular plan based on specific experiential data for each subgroup covered by the plan. The period that an actuary analyses is usually 3 to 5 years. The analysis covers employees and beneficiaries older than 50[11]. Namely, the minimum requirement for pension is observed (minimum age and service requirements).

[11] IRS recommendation.

To compile a mortality table, it is necessary to calculate death probability $q_x$ for each age class. As stated in the previous chapter, the analysis needs to include a sufficient number of deaths, namely, 1.082 of deaths in each age class to achieve full credibility (assuming that r = 5% and p = 90%). For example, if $q_{72}$ = 0,031, it takes 1,082 / 0,031 = 34.903 live persons aged 72 to achieve full credibility of mortality rate at such age, which is a large quantity of data. In addition, the obtained values of qx must then be levelled, which requires additional analyses and approximations. Therefore, in most cases it is more acceptable to fit standard mortality tables to a specific subset of the insured.

The first step in adjusting mortality rates is to find the weight $\hat{\xi}$ which represents the ratio between actual deaths and expected number of deaths from all age groups. The ratio is applied to all insured persons aged x, and is determined as an aggregate value through all age groups,

$$\hat{\xi} = \frac{\sum_x d_x}{\sum_x q_x^{st}},$$

where mortality $q_x^{st}$ is obtained from standard mortality tables.

Full credibility is achieved when the ratio $\hat{\xi}$ is within the margin of error (r) of actual ratio $\xi$, with probability of at least p:

$$\Pr\left(\left|\hat{\xi} - \xi\right| \le r\xi\right) \ge p.$$

Standard credibility formula is constructed as follows

$$q_x^{est} = Z \cdot \left(\hat{\xi} \cdot q_x^{st}\right) + \left(1 - Z\right) \cdot q_x^{st} = \left(Z \cdot \hat{\xi} - Z + 1\right) \cdot q_x^{st},$$

where $q_x^{est}$ is estimated death rate of a plan.

## III Numerical Example

Let us assume that a three-year data on women who are the members of a pension scheme show that there were 1,230 deaths (Table 1). The actuary will need to modify mortality rates shown in the standard mortality tables to the population of women in Serbia (Mortality Tables of the Statistical Office of the Republic of Serbia 2010-2012) and generate new mortality tables adjusted to the particular plan, assuming that the confidence interval is 95% and the margin of error is 5%.

According to the Normal Distribution tables $N\left(0,1\right)$ for p = 0,95 value is $z_\alpha = 1,96,$ whereas for the margin of error of 5% full credibility is reached for $\left(1,96 / 0,05\right)^2 = 1.537$ deaths. As the number of actual deaths is 1,230, this number is not sufficient for full credibility and therefore, the credibility factor is calculated as the square root of the ratio of the actual deaths to the number of deaths required for full credibility $Z = \sqrt{\frac{1.230}{1.537}} = 0,895.$

Expected number of deaths is 2,097 (column 6). Further, the weight $\hat{\xi}$ is calculated which represents the ratio of the actual deaths to the expected number of deaths for all age groups ' $\hat{\xi} = 1.230 / 2.097 = 0,59$, and thus

Weight = 0.895 · 0,59 + (1 – 0.895) · 1 = 0,63 (column 8).

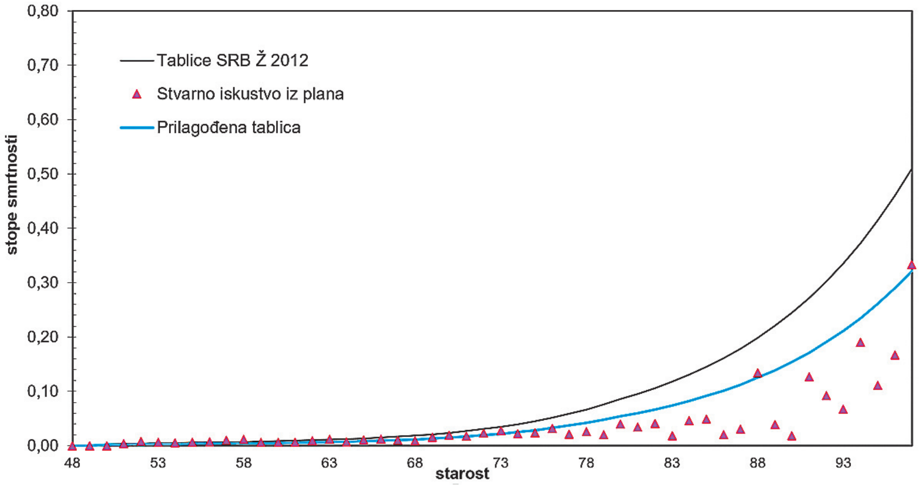
The calculation shows that every mortality rate $q_x^{st}$ from the standard mortality table needs to be adjusted with the weight 0.63 in order to obtain the mortality rate $q_x^{est}$ estimated for the plan.

*Table 1 Formation of adjusted mortality table*

| x | lx | $d_x$ | $q_x^{st}$ | $q_x^{\exp}$ | Expected number of deaths | Adjusted rate $q_x^{est}$ | Weight | Ratio |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5=2/4 | 6=2*5 | 7 | 8=7*2/6 | 9=3/6 |
| 50 | 503 | | 0,003455 | - | 1,739287 | 0,002177 | 0,630 | - |
| 51 | 891 | 3 | 0,003738 | 0 ,003922 | 3,332705 | 0,002355 | 0,630 | 1,049 |
| 52 | 1.484 | 12 | 0,004224 | 0,008245 | 6,268780 | 0,002661 | 0,630 | 1,952 |
| 53 | 2.192 | 14 | 0,004653 | 0,006380 | 10,198316 | 0,002931 | 0,630 | 1,371 |
| 54 | 2.459 | 12 | 0,005171 | 0,004975 | 12,718530 | 0,003258 | 0,630 | 0,962 |
| 55 | 2.858 | 19 | 0,005730 | 0,006728 | 16,375410 | 0,003610 | 0,630 | 1,174 |
| 56 | 3.540 | 24 | 0,006180 | 0,006914 | 21,874295 | 0,003893 | 0,630 | 1,119 |
| 57 | 3.655 | 37 | 0,006681 | 0,010043 | 24,418500 | 0,004209 | 0,630 | 1,503 |
| 58 | 3.493 | 42 | 0,007267 | 0,012012 | 25,380635 | 0,004578 | 0,630 | 1,653 |
| ⋮ | | | | | | | | |
| 93 | 26 | 2 | 0,335770 | 0,066667 | 8,803892 | 0,211535 | 0,630 | 0,199 |
| 94 | 5 | 1 | 0,372676 | 0,190694 | 1,954310 | 0,234786 | 0,630 | 0,512 |
| 95 | 16 | 2 | 0,413793 | 0,111111 | 6,509793 | 0,260690 | 0,630 | 0,269 |
| 96 | 10 | 2 | 0,460268 | 0,166667 | 4,827294 | 0,289969 | 0,630 | 0,362 |
| 97 | 10 | 3 | 0,510516 | 0,333333 | 5,354294 | 0,321625 | 0,630 | 0,653 |
| Total | 1.358 | 1.230 | | | 2.097 | | | |

A graphical representation of mortality tables, that is, mortality rates, is useful before and after adjusting the tables. One way to assess whether an adjustment is appropriate is to compare the actual mortality rates with the corresponding standard mortality table, as shown in the Chart 1. It can be seen that after the rate correction for the calculated value of 0.63, the shape of the curve is similar to the one obtained from the standard mortality table. It can be also noted that after the age of seventy, the deviations of the actual rates of the observed plan are higher than those of the standard mortality tables and therefore, in this example, it is justified to apply the presented method.

*Image 1. Comparative view of mortality tables*




## IV Conclusion

Credibility theory is nowadays widely used in life and non-life insurance. The main goal of applying the theory of credibility is to minimise the errors between the statistical evaluation of various parameters and their actual value. Credibility theory enables the application of different methods and models in the estimation of certain elements of an observed population subgroup, combining the outputs for such a particular subgroup with those obtained for the population as a whole. In contemporary actuarial life insurance practice, credibility serves as a tool for adapting standard mortality tables to specific retirement plans, that is, to the population covered by those plans. When evaluating a retirement plan, credibility theory allows the incorporation of experiential data of participants covered by the evaluated plan. Various models of credibility theory are used to modify the standard population mortality tables in order to implement quantitative experiences related to population subgroups, which contributes to increasing the reliability of estimates. In a highly competitive insurance market, a sound knowledge of credibility theory is an essential tool in assessing insurance risk.

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