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CONSTRUCTION OF C-CREDIBILITY MEASURE AND APPLICATION IN INSURANCE

SCIENTIFIC PAPER

Abstract

Credibility theory provides a tool for modelling various phenomena in insurance. Changes in life circumstances such as the pandemic and its consequences require adjustment of old and creation of new theories that would improve existing parameters in modelling phenomena in insurance and actuarial analysis. Thus, by using c-credibility measure, we try to respond to some of the challenges that have emerged. The paper presents a summarised theory of c-credibility and gives an algorithm for constructing c-credibility measure, which is illustrated by an example.

Key words: *equilibrium, fuzzy complement, c-credibility measure*

I. Introduction

Insurance connects the theory of various scientific disciplines and practice, thus making actuarial science multidisciplinary in its combination of science and practical application. Credibility theory and its methods have proven very useful in many actuarial analyses and have been applied since the very beginnings of actuarial development. The first studies dealt with estimates of the mean value of the frequency of claims using classical and empirical credibility procedures.[2] Later, various authors developed credibility formulas, mainly for claims, including Bayesian credibility procedures. However, the concepts of credibility are also used in other actuarial activities so that today, credibility can be used to determine the price of insurance coverage, calculate the insurance premium rate, determine the future premium rate based on experience and provisions, adjust mortality tables, etc.

Credibility theory is based on mathematical or statistical fundamentals and reliable data. Credibility provides tools for dealing with randomness of variables i.e.

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[2] Albert H. Mowbray, *How extensive a payroll exposure is necessary to give a dependable pure premium?*, Proceedings of the Casualty Actuarial Society, 1914.

data that are used for predicting future events. The Actuarial Standard of Practice (ASOP 25[3]) provides guidance to actuaries when performing professional services with respect to selecting or developing credibility procedures and the application of those procedures to the sets of data. This Standard provides definitions of terms used in credibility, as well as recommended practices for the selection, development, or use of credibility and actuarial valuation. The subjectivity of actuaries in the assessment of certain parameters is inevitable, due to lack of data, uncertainty of certain tasks, model requirements, but also a high degree of complexity of the analysed problem. This is why the methods of fuzzy mathematics are increasingly used in various segments of insurance theory and practice.

In recent years, insurance companies and various funds have paid more attention to new phenomena that are happening, such as Covid-19, which in turn are reflected in insurance (increased mortality of the elderly), for example in pension schemes. Variability in mortality rates within different demographic groups and/or populations within the schemes results in the need to reconsider assumptions and better adapt to specific plans. This has led to an interest in new theories such as for example, classical credibility theory and also the latest c-credibility theory as the means of adapting standard mortality tables to specific pension schemes or to the population covered by those schemes.[4]

The section two introduces new terms of fuzzy complement and its equilibrium. Properties related to them are also listed, all for the purpose of axiomatically establishing a measure of c-credibility. Unlike a much better known measure of a set of events - a probability that is additive, generally, this is not the case with this measure. It belongs to the class of so-called non-additive measures, that is, fuzzy measures. It depends not only on the set under consideration but also on the mentioned fuzzy complement and the appropriate parameters which enable a good selection to achieve the desired accuracy of the model. In addition, particular properties of this measure are listed, of which the extension theorem is the most suitable for application. The section three provides an algorithm for constructing c-credibility measure based on the extension theorem. The whole proposed algorithm is illustrated by an example. The section four provides concluding observations.

II. C-credibility Measure

Defining c-credibility measure requires a preliminary explanation of the necessary terms.

Function $c:[0,1]\rightarrow[0,1]$ is fuzzy *complement* provided that the following conditions are met:

c1) $c(0)=1$, $c(1)=0$ (boundary condition)

[3] Actuarial Standard of Practice No. 25. *Credibility Procedures Applicable to Accident and Health, Group Term Life and Property/Casualty Coverage.*

[4] See the paper of Marija Paunović, Vladimir Gajović, (2020). Adjustment of Mortality Tables by Limited Fluctuation Method, *Insurance Trends 36* (1), 16–24.

c2) $(\forall a, b \in [0,1])\ a \leq b \Rightarrow c(a) \geq c(b)$ (monotonicity).

If for all $a \in [0,1]$ from [0,1] $c(c(a)) = a$, then complement *c is involutive*.

If *c* is a continuous function, then we say that *c is a continuous* fuzzy complement.

If $c:[0,1] \to [0,1]$ is an involutive, monotonic and non-increasing function, than follows that *c* is a continuous bijective function for which boundary conditions are valid.

The equilibrium of fuzzy complement is element $\varepsilon \in (0,1)$ such that $c(\varepsilon) = \varepsilon$.

Every fuzzy complement has at most one equilibrium. If *c* is a continuous fuzzy complement, then *c* has a unique equilibrium.

As examples of important continuous involutive fuzzy complements we will present standard, Sugeno and Yager fuzzy complement and their equilibriums:

1) $c(a) = 1 - a$, $\varepsilon = \frac{1}{2}$;

2) $c_\lambda(a) = \frac{1-a}{1+\lambda a}$, $\lambda > -1$, $\varepsilon = \frac{\sqrt{1+\lambda} - 1}{\lambda}$ $(\lambda \neq 0)$, $\varepsilon = \frac{1}{2}$ $(\lambda = 0)$;

3) $c_\lambda(a) = (1 - a^\lambda)^{1/\lambda}$, $\lambda > 0$, $\varepsilon = \left(\frac{1}{2}\right)^{1/\lambda}$.

Let *P*(Ω) be a partitive, nonempty set Ω. Its each element will be called an event. For the axiomatic definition of credibility, it is necessary to assign the number *cr (A)* to each event *A*, that is, the credibility that event *A* will occur, so that such an association is a set function with corresponding properties.

Let $c:[0,1] \to [0,1]$ be an involutive fuzzy complement with equilibrium ε. *The c-credibility measure* on Ω (determined with *c*) is a set function $cr : P(\Omega) \to [0,1]$ such that

cr1) $cr(\emptyset) = 0$;

cr2) $(\forall A, B \in P(\Omega))\ A \subset B \Rightarrow cr(A) \leq cr(B)$;

cr3) $(\forall A \in P(\Omega))\ cr(\overline{A}) = c(cr(A))$;

cr4) $cr\left(\bigcup_{j \in J} A_j\right) = \sup_{j \in J} cr(A_j)$,

for arbitrary sets $A_j \in P(\Omega)$, $j \in J$, for which $\sup_{j \in J} cr(A_j) < \varepsilon$, where *J* is an arbitrary index set. The triplet $(\Omega, P(\Omega), cr)$ is called *c-credibility space*.

Example 1 Set function $cr : P(\Omega) \to [0,1]$, $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, defined with $cr(\emptyset) = 0$, $cr(\Omega) = 1$, $cr(\{\omega_1\}) = 0.1$, $cr(\{\omega_2\}) = 0.3$, $cr(\{\omega_3\}) = 0.4$, $cr(\{\omega_4\}) = 0.6$, $cr(\{\omega_1, \omega_2\}) = 0.3$, $cr(\{\omega_1, \omega_3\}) = 0.4$,

$cr(\{\omega_2,\omega_3\})=0.4,\ cr(\{\omega_1,\omega_4\})=0.6,\ cr(\{\omega_2,\omega_4\})=0.6,\ cr(\{\omega_3,\omega_4\})=0.7,$ $cr(\{\omega_1,\omega_2,\omega_3\})=0.4,\quad cr(\{\omega_1,\omega_2,\omega_4\})=0.6,\quad cr(\{\omega_1,\omega_3,\omega_4\})=0.7,$ $cr(\{\omega_2,\omega_3,\omega_4\})=0.9,$ is c-credibility measure relative to standard fuzzy complement.

Credibility is regular fuzzy measure i.e. cr(Ω) = 1. Additionally, 0 ≤ *cr*(*A*) ≤ 1 for any subset *A* of Ω. This measure is subadditive, namely (∀*A*,*B* ∈P(Ω)) *cr*(*A* ∪*B*) ≤ *cr*(*A*) + *cr*(*B*), if *c* is involutive fuzzy complement such that $c(a)\geq 1-a,$ for all $a\in[0,1]$. It is additive if and only if there are at most two singletons in P(Ω) taking nonzero credibility values. It is also semicontinuous. Extension theorem is very important for this measure:

If $\Omega\neq\varnothing$ and $cr:P(\Omega)\to[0,1]$ is c-credibility measure, then so-called extension conditions apply

$$\sup_{\omega\in\Omega} cr(\{\omega\})\geq\varepsilon \tag{1}$$

$$cr(\{\omega^*\})\geq\varepsilon \;\Rightarrow\; \sup_{\omega\neq\omega^*} cr(\{\omega\})=c(cr(\{\omega\})). \tag{2}$$

The following extension theorem allows the calculation of numerical values of the c-credibility measure for any event, based on the credibility values of each individual singleton, and it provides a sufficient condition for the c-credibility measure.

Theorem 1. Let $c:[0,1]\to[0,1]$ be an involutive fuzzy complement, whose equilibrium is ε, and $cr:\{\{\omega\}|\omega\in\Omega\}\to R_0^+,\quad \Omega\neq\varnothing$ set function which satisfies (1) and (2). Then *cr* has a unique extension on c-credibility measure $cr:P(\Omega)\to R_0^+$ defined with

$$cr(A)=\begin{cases}\sup\limits_{\omega\in A} cr(\{\omega\}), & \sup\limits_{\omega\in A} cr(\{\omega\})<\varepsilon\\ c\left(\sup\limits_{\omega\in \underline{A}} cr(\{\omega\})\right), & \sup\limits_{\omega\in A} cr(\{\omega\})\geq\varepsilon\end{cases} \tag{3}$$

for nonempty set *A*, and *cr*(∅) = 0.

III. Algorithm and Numerical Example

Let us assume that an insurance company offers a number of its services, i.e. insurance package, that is, let a set of elementary events be $\Omega=\{\omega_1,\omega_2,\ldots,\omega_k\}$. Naturally, the company may offer more complex packages (certain subsets *A* of Ω), complete range *A*=Ω or may not offer anything, i.e. $A=\varnothing$. We wish to ascertain the credibility of each event, but so as to obtain c-credibility measure *cr*. Therefore, firstly, the value of credibility for each event A is the number from the interval [0.1], the number of complete range (safe event) Ω is 1, and of the impossible event $\varnothing$ is 0. Naturally, monotonicity must be meet, namely, a larger event is assigned a larger credibility. According to the credibility extension theorem (on the representation of *cr* over singleton credibility – sets of single elements), it would be necessary to set the singleton credibility measures. For easier presentation of the example, we will assume that $cr(\{\omega_1\}) \leq cr(\{\omega_2\}) \leq \ldots \leq cr(\{\omega_k\})$. The offer should be such that only one singleton (in our case $\{\omega_k\}$) may have the credibility higher than, or equal to a ε <1, whereas others ($\{\omega_1\}$,…,$\{\omega_{k-1}\}$) must have the credibility less than ε. Offer (service) $A = \{\omega_{i_1}, \omega_{i_2}, \ldots, \omega_{i_r}\}$, $i_1 < i_2 < \ldots < i_r$ created out of them has the same credibility as singleton with the highest credibility, i.e. $cr(A) = cr(\{\omega_{i_r}\})$. If $\omega_k \in A$, then from the elements from $\overline{A}$ we choose the element whose singleton has the highest credibility and the complement of such credibility is the credibility of *A*. In addition, the second condition of the extension should be met: that the highest of all singleton credibilities $\{\omega_1\}$,…,$\{\omega_{k-1}\}$ must be equal to the credibility complement for $\{\omega_k\}$. Credibility is further assigned to the remaining singletons. The procedure is then continued for the sets of multiple elements, based on the extension theorem.

C-credibility measure is appropriate because we independently choose fuzzy complement and its equilibrium. The professional i.e. the actuary chooses which elementary events (packages) are such that by increasing the offer of such packages we do not exceed the credibility of the largest elementary package, but remain on it. The credibility values of such packages do not exceed the equilibrium, so even in that way we can arrive at an estimate of the equilibrium value.

Let us illustrate the above in the following example. Fort the set $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ we determine set function $cr : P(\Omega) \rightarrow [0,1]$, so that it is c-credibility. Naturally, it must be $cr(\varnothing) = 0,\ cr(\Omega) = 1.$

Firstly we assess the singleton with the highest credibility, for example $cr(\{\omega_4\}) = 0.6.$

Then we choose a fuzzy complement, e.g. Sugeno class fuzzy complement and let its equilibrium be $\varepsilon = 0.4 < 0.6$. Selection of complement from the Sugeno class complements depends on the selection of parameter λ.

Equilibrium determines the value of that parameter: $\lambda = \frac{1-2\varepsilon}{\varepsilon^2} = \frac{5}{4} = 1.25$.

Therefore, c-credibility measure is defined in relation to the fuzzy complement $c(a) = \frac{1-a}{1+1.25a}$. Now we assess credibility of other singletons so that they are smaller than the equilibrium. Further, it can be inferred from the extension conditions that

$$cr(\{\omega_3\}) = \max(cr(\{\omega_1\}),\ cr(\{\omega_2\}),\ cr(\{\omega_3\})) =$$
$$c(cr(\{\omega_4\})) = c(0.6) = \frac{1-0.6}{1+1.25\cdot 0.6} = \frac{8}{35} \approx 0.229.$$

Let us say that $cr(\{\omega_1\}) = 0.1,\ cr(\{\omega_2\}) = 0.2.$ Using (3) we calculated the credibility of multielement sets:

$$cr\left(\{\omega_1,\omega_2\}\right) = \max(cr\left(\{\omega_1\}\right), cr\left(\{\omega_2\}\right)) = 0.2,$$
$$cr\left(\{\omega_1,\omega_3\}\right) = \max(cr\left(\{\omega_1\}\right), cr\left(\{\omega_3\}\right)) \approx 0.229,$$
$$cr\left(\{\omega_2,\omega_3\}\right) = \max(cr\left(\{\omega_2\}\right), cr\left(\{\omega_3\}\right)) = 0.229,$$
$$cr\left(\{\omega_1,\omega_4\}\right) = c(\max(cr\left(\{\omega_2\}\right), cr\left(\{\omega_3\}\right))) = c(0.229) \approx 0.599,$$
$$cr\left(\{\omega_2,\omega_4\}\right) = c(\max(cr\left(\{\omega_1\}\right), cr\left(\{\omega_3\}\right))) = c(0.229) \approx 0.599,$$
$$cr\left(\{\omega_3,\omega_4\}\right) = c(\max(cr\left(\{\omega_1\}\right), cr\left(\{\omega_2\}\right))) = c(0.2) = 0.64,$$
$$cr\left(\{\omega_1,\omega_2,\omega_3\}\right) = \max(cr\left(\{\omega_1\}\right), cr\left(\{\omega_2\}\right), cr\left(\{\omega_3\}\right))) \approx 0.229,$$
$$cr\left(\{\omega_1,\omega_2,\omega_4\}\right) = c(cr\left(\{\omega_3\}\right)) = c(0.229) \approx 0.599,$$
$$cr\left(\{\omega_1,\omega_3,\omega_4\}\right) = c(cr\left(\{\omega_2\}\right)) = c(0.2) = 0.64,$$
$$cr\left(\{\omega_2,\omega_3,\omega_4\}\right) = c(cr\left(\{\omega_1\}\right)) = c(0.1) = 0.8\,.$$

IV. Conclusion

Today, credibility theory is extensively used in insurance. It can be said that the main goal of applying the classical theory of credibility is to minimize the errors between the statistical evaluation of various parameters and their actual value. Credibility theory enables the application of different methods and models when estimating the value of certain elements of a subset of the observed population, by combining the outputs for such specific subset with the results obtained for the population as a whole. Different models of credibility theory, including c-credibility, are used for modelling many problems in insurance, which increases the parameter assessment reliability. In the conditions of heightened competition on the insurance market, every new theoretical method, including the theory of c-credibility as an important tool for insurance risk assessment, is of vital importance.

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